Material Quantification Using Spectral Computed Tomography

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***Abstract*—The aim of this work is to evaluate a linear algebra technique for materials quantification using spectral Computed Tomography (CT). The MARS spectral micro-CT system incorporating Medipix3 was used to acquire spectroscopic CT data from phantoms containing (i) calcium chloride solutions of various concentrations and (ii) sunflower oil and discrete solutions of iodine, ferric nitrate and calcium chloride. These data were used to establish the linearity of the system and to calibrate the spectroscopic response for different materials of interest. The validity of the proposed materials analysis method was determined by analysis of the information entropy and degrees of freedom associated with the inverse calibration matrix. It is concluded that materials analysis is viable using the proposed linear algebra method for some of the materials of interest and the efficacy of the method is improved with the use of appropriate volume constraints. However the method may not be able to independently distinguish iron, calcium, oil and water without additional data and/or constraints.**

***Index Terms*—linear algebra, entropy, spectral CT, efficacy, material quantification, constraint**

I. Introduction

Spectral CT is a new trend in x-ray CT that expands the monochromatic nature of standard CT to multiple energies. It is anticipated that the spectroscopic information will be useful for improving contrast resolution and the quantification of material composition [1]-[5].

Conventional CT based on a single photon energy range has limited value in this respect. With knowledge of how x-rays interact with materials, the new development of energy resolving photon-counting detectors allow material information to be extracted from spectral CT data, thus enabling material differentiation.

The potential to use spectral information from x-ray beams was first reported in the mid 1970s. The early work focused on material decomposition of the linear attenuation coefficient into contributions from the photoelectric process and the Compton effect with two spectrally distinct measurements [6].

In recent years, the development of spectroscopic photon-counting detectors have opened up new possibilities for improving material-specific imaging with spectral (multi-energy) CT. The development of spectral CT promises to enable exciting new research and ultimately provide improved clinical diagnostic capability based on quantitative tissue characterization The signal output in integrating detector is dependent on the energy flux integrated over the entire x-ray spectrum.

Therefore, no element-specific attenuation profiles with characteristic photon energy distributions can be obtained. In contrast, spectral CT with the use of energy resolving photon-counting detectors is capable of extracting quantitative information about the elemental, molecular information of tissues and contrast materials on the basis of their attenuation properties [7]-[11].

The MARS research team in Christchurch, New Zealand has developed a spectral micro-CT system equipped with a Medipix3 photon-counting detector that will be used by researchers to study advanced imaging techniques for health research applications. In particular, researchers aim to investigate spectroscopic methods for quantifying the fat, calcium and iron components of soft-tissues within small animal and specimen studies of diseases such as fatty-liver and arterial atherosclerosis [12]-[18].

At energies relevant to this work, x-rays interact predominantly by a combination of the photoelectric and Compton effects. The photoelectric effect varies with material density and atomic number according to $\sim\rho Z^4$. The Compton effect varies according to $\sim\rho Z$. Both effects are energy dependent and thus material decomposition becomes possible with the acquisition of CT data at multiple energies [5].

The purpose of this study is to evaluate a linear algebra technique for materials quantification. The MARS spectral micro-CT scanner has been calibrated experimentally with phantoms containing known solutions of clinically relevant materials and the resulting spectroscopic CT data analyzed to determine the feasibility of the proposed analysis method.

A. Theory

CT numbers are effective linear attenuation values measured in reconstructed voxels and scaled so that by definition CT_{air} = -1000 and CT_{water} = 0. We can transform the CT number measurements into values that

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are directly proportional to the attenuation coefficients of the unknown materials. For $\mu_{air} << \mu_{water}$ we obtain:

$$CT = 1000(\mu - \mu_{water})/(\mu_{water} - \mu_{air}) \quad (1)$$

In cases where beam-hardening and scattering are negligible Beer's Law applies and the effective linear attenuation coefficient of a mixture of material components x_i, $i = 1\ldots M$ is the sum of the attenuation coefficients for the individual components.

$$z = (\sum_i \mu_i x_i) / \mu_{water} \quad (2)$$

Measuring the CT numbers of a set of material components at multiple energy thresholds $j = 1\ldots N$ allows us to write a linear matrix equation for the transformed variables $[z_j]$ in terms of the (unknown) composition $[x_i]$ and a spectroscopic calibration matrix **A** with errors ε which we will assume follow a Gaussian distribution:

$$\mathbf{z} = \mathbf{A} \bullet \mathbf{x} + \epsilon \quad (3)$$

If necessary, constraints can be built into the expression above by augmenting it with additional linear equations. From an initial estimate of the composition $\mathbf{x_0}$ and its covariance $\mathbf{P_0}$, the optimal solution [6] for the updated composition $\mathbf{x_1}$ and covariance $\mathbf{P_1}$ given a measurement **z** with covariance $\mathbf{C_z}$ is:

$$\mathbf{P_1} = (\mathbf{A}^T \mathbf{C_z}^{-1} \mathbf{A} + \mathbf{P_0}^{-1})^{-1} \quad (4)$$

$$\mathbf{x_1} = \mathbf{x_0} + \mathbf{P_1} \cdot \mathbf{A}^T \mathbf{C_z}^{-1} (\mathbf{z} - \mathbf{A} \cdot \mathbf{x_0}) \quad (5)$$

The effectiveness of the measurement may be measured by the reduction in information entropy δH of the covariance matrix $\mathbf{P_1}$ and by the increase in the effective number of degrees of freedom (d.f.) of the system. The information entropy is a logarithmic measure of the volume of the uncertainty covariance with change after measurement given by:

$$\delta H = 1/2\ (\log(\det(\mathbf{P_1})) - \log(\det(\mathbf{P_0}))) \quad (6)$$

The effective d.f. for the fitted composition measures the number of components recovered by the measurement and is given by the trace of the *hat* matrix:

$$v = \mathrm{tr}\ (\mathbf{A}^T \mathbf{P_1} \mathbf{A}^T \mathbf{Cz}^{-1}) \quad (7)$$

II. Materials and Methods

A. Spectral CT Systems

The MARS-CT scanner is currently being operated in the Bioengineering Lab of the Department of Radiology at Christchurch Hospital. It comprises a micro-focus x-ray tube (Source-Ray SB-80-1K) and a Medipix3 detector within a rotating gantry driven by computer-controlled stepper motors.

The Medipix 3 photon counting detector is bump bonded onto a 300 μm thick silicon sensor layer and configured in 55 μm *fine-pitch mode*. The CT gantry has the facility for providing variable magnification via movable camera and source platforms. For this work the source-to-detector and source-to-object distances were held constant to provide a magnification factor of 1.44 and a CT reconstruction voxel size of $(38\ \mu m)^3$.

B. Linearity

A perspex phantom of 10 mm diameter containing aqueous calcium chloride solutions of various concentrations 74.92, 147.20, 294.30 and 735.10 $mg.ml^{-1}$ was used to evaluate the linearity of the system response with calcium concentration.

It was scanned at 50 kVp with a detector threshold energy of 12 keV. The CT number and standard deviation for each concentration were measured in Hounsfield units (HU) for an ROI placed manually over each solution insert. The relationship between CT number and calcium chloride concentration was determined by linear regression analysis.

C. Spectroscopy

A second perspex phantom of 10 mm diameter containing ferric nitrate, calcium chloride, sunflower oil, iodine, air and water was used to evaluate the spectroscopic CT number response for different materials. The phantom was scanned at 50 kVp with detector threshold energies of 9.8, 15.1, 20.4, 25.6, 30.9 and 36.2 keV.

The CT number and standard deviation for each material were measured in HU for an ROI placed manually over each solution insert. The relationship between CT number and detector threshold energy was evaluated graphically.

D. Material Analysis

The spectroscopic CT number response for each material was transformed as described above to be proportional to the material attenuation coefficient. For sunflower oil and water the response was expressed in terms of a unit volume of the liquid at room temperature. For the dissolved species (Ca, Fe, I) we expressed the response in terms of the attenuation of the species alone.

$$A_{ij} = (CT_{ij} / 1000) + 1 \text{ for water and oil} \quad (8)$$

$$A_{ij} = (1 / C)\ (CT_{ij} / 1000) \text{ for dissolved species} \quad (9)$$

where C is the molar concentration of the dissolved species.

Response matrices were assembled for measurement scenarios with different sets of unknowns: (i) {Ca, Fe, Oil, Water}; (ii) {I, Oil, Water}; (iii) {Oil, Water}. We also included an optional volume conservation constraint: $V_{oil} + V_{water} = constant$.

The change in information entropy and d.f. obtained were calculated for measurements using up to six threshold energies for both the constrained and unconstrained systems of equations. The initial state of uncertainty (covariance matrix) before measurement was defined to be $\mathbf{P_0} = \mathbf{I}$.

For each scenario the expected covariance matrix after measurement $\mathbf{P_1}$ was calculated using the appropriate response matrix **A** and an input data covariance matrix $\mathbf{C_z}$ computed assuming standard measurement uncertainties

of 10 HU on the CT number data. The change in information entropy after a set of measurements was computed from the determinant of the updated covariance matrix $\mathbf{P_1}$:

$$\delta H = 1/2 \log (\det (\mathbf{P_1})) \quad (10)$$

The d.f. of the system after a set of measurements was computed from the trace of the *hat* matrix as described previously.

III. Results and Discussion

A. Linearity

Fig. 1 and Fig. 2 show a CT reconstruction of the linearity phantom and the graph of CT number as a function of increasing calcium chloride concentration. The result of the linear regression shows that the system is linear over the range of concentrations to at least 294.30 mg.ml^{-1} with R^2 of 0.998.

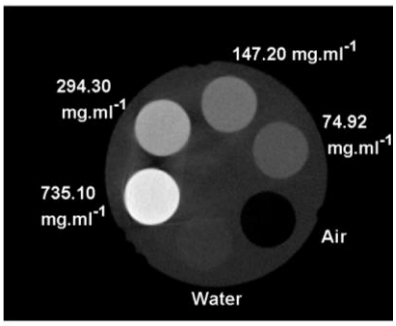


Figure 1. Reconstructed CT image of perspex phantom with solutions of different concentrations of calcium chloride acquired with a detector threshold energy of 12 keV and a tube voltage of 50 kV.

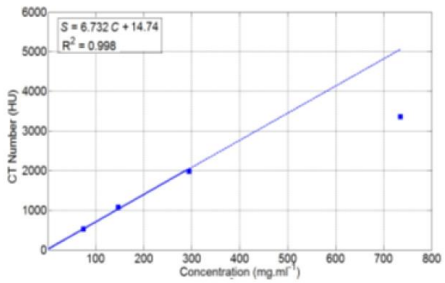


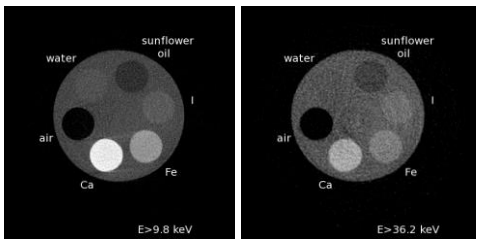
Figure 2. The measured CT numbers as a function of concentration. The standard uncertainties for each data point are ~10 HU.

It is not linear up to the highest concentration of calcium chloride (735.10 mg.ml^{-1}). The linearity measurements indicate that the system response is linear up to ~300 mg.ml^{-1} of calcium chloride corresponding to a CT number of ~2000 HU.

Examination of the CT images near regions of high calcium concentration shows evidence of the beam-hardening effect that limits the linearity of the system at higher concentrations.

B. Spectroscopy

The CT reconstructions for the lowest and highest detector energy thresholds together with the CT spectra across all energies for each material are shown in Fig. 3. The CT numbers for sunflower oil and iodine increase with detector threshold energy, whereas those for calcium chloride and ferric nitrate decrease with energy (Fig. 4). The spectroscopic response to the materials tested (Ca, Fe, I, sunflower oil and water) is consistent with the energy dependent attenuation coefficients for those materials.



(a) (b)

Figure 3. Spectral CT reconstructions of a perspex phantom containing calcium chloride, ferric nitrate, iodine, sunflower oil, water and air. (a) Data were acquired with a tube voltage of 50 kVp and threshold energies of 9.8 and (b) 36.2 keV.

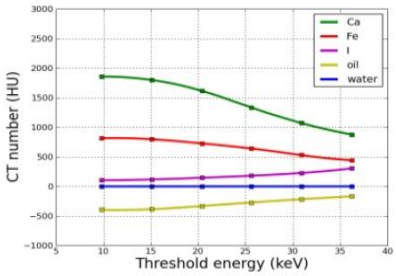


Figure 4. The spectral CT numbers for each material plotted for a range of detector threshold energies. The standard uncertainties for each data point are in the range of 3-12 HU.

At low energies the CT number response for the higher-Z materials is much greater than that of water due to the strong influence of the photoelectric effect ($\mu \sim \rho Z^4$) at these energies.

With increasing energy the CT numbers for these materials decrease as the relative contribution of the photoelectric effect reduces and Compton scattering becomes more significant. The CT number for iodine increases with threshold energy due to the influence of the K-edge at 33.2 keV [7].

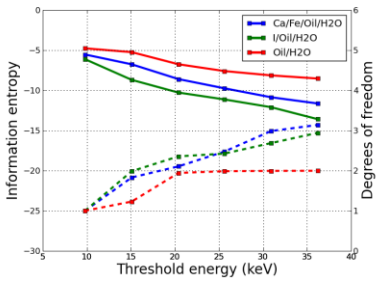
C. Material Analysis

The cumulative information entropy and degrees of freedom (d.f.) after a sequence of up to six measurements at different threshold energies for both the unconstrained and constrained systems of equations are given in Fig. 5.

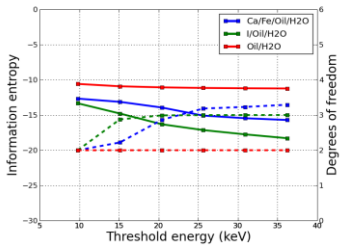
The unconstrained measurements of {Oil, Water} recover two d.f. after the first three spectroscopic measurements have been made. The full complement of d.f. for the {I, Oil, Water} case are recovered only after six spectroscopic measurements.

For the constrained system, the full d.f. for these two scenarios are quickly recovered after only the first and third spectroscopic measurements respectively. The full d.f. for {Ca, Fe, Oil, Water} are never recovered with either system of equations. In all cases the information

entropy is significantly improved by the use of volume constraints.



(a)



(b)

Figure 5. (a) The cumulative information entropy (solid lines) and d.f. (dashed lines) retrieved by a sequence of spectral CT measurements for an unconstrained system; (b) The cumulative information entropy and d.f. for a system including volume constraints on oil and water.

The linear method for materials analysis performs well for up to three materials, especially when using the additional volume constraint. For the case of four material unknowns (Ca, Fe, sunflower oil and water) the analysis is unable to return the full number of d.f. even with the use of six energy thresholds.

To distinguish both Fe and Ca in the presence of oil and water may require further work to investigate the impact of additional energy thresholds and/or alternative constraints such as mass conservation.

IV. Conclusion

Material analysis is viable using the proposed linear algebra method for a number of material combinations and the efficacy of the method is improved with the use of appropriate volume constraints.

However the method may not be able to independently distinguish Fe, Ca, oil and water without additional data or constraints.

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References

[1] J. P. Ronaldson, R. Zainon, N. J. Scott, S. P. Gieseg, A. P. Butler, P. H. Butler, *et al*., "Towards quantifying the composition of soft-tissues by spectral CT imaging with Medipix3," *Medical Physics*, vol. 39, pp. 6847-6857, Nov. 2012.

[2] M. A. Hurrell, A. P. Butler, N. J. Cook, P. H. Butler, J. P. Ronaldson, and R. Zainon, "Spectral Hounsfield Units - A new radiological concept," *European Radiology*, vol. 22, pp. 1008-1013, May 2012.

[3] R. Zainon, J. P. Ronaldson, T. Janmale, *et al*., "Spectral CT of carotid atherosclerotic plaque: Comparison with histology," *European Radiology*, vol. 22, pp. 2581-2588, Dec. 2012.

[4] P. He, H. Yu, J. Bennett, P. Ronaldson, R. Zainon, A. Butler, *et al.*, "Energy-discriminative performance of a spectral micro-CT system," *Journal of X-ray Science and Technology*, vol. 21, pp. 335-345, 2013.

[5] C. D. Rodgers, "Information content and optimisation of high spectral resolution measurements," in *Proc. SPIE*, United States, 1996, pp. 136-147.

[6] R. E. Alvarez, and A. Macovski, "Energy-selective reconstructions in x-ray computerised tomography," *Physics in Medicine and Biology*, vol. 21, pp. 733-744, 1976.

[7] J. P. Schlomka, E. Roessl, R. Dorscheid, S. Dill, G. Martens, T. Istel, *et al*., "Experimental feasibility of multi-energy photon counting k-edge imaging in pre-clinical computed tomography," *Physics in Medicine and Biology*, vol. 53, pp. 4031-4047, 2008.

[8] X. Liu, L. Yu, A. Primak, and C. McCollough, "Quantitative imaging of element composition and mass fraction using dual-energy CT: Three-material decomposition," *Medical physics*, vol. 36, pp. 1602-1609, May 2009.

[9] N. G. Anderson, A. P. Butler, and N. J. Scott, *et al.*, "Spectroscopic (multi-energy) CT distinguishes iodine and barium contrast material in MICE," *European Radiology*, vol. 20, pp. 2126-2134, 2010.

[10] M. Aslund, E. Fredenberg, M. Telman, and M. Danielsson, "Detectors for the future x-ray imaging," *Radiation Protection Dosimetry*, vol. 139, pp. 327-333, 2010.

[11] P. M. Shikhaliev, "Computed tomography with energy-resolved detection: A feasibility study," *Physics in Medicine and Biology*, vol. 53, pp. 1475-1495, 2008.

[12] P. M. Shikhaliev and S. G. Fritz, "Photon counting spectral CT versus conventional CT: Comparative evaluation for breast imaging application," *Physics in Medicine and Biology*, vol. 56, pp. 1905-1930, 2011.

[13] R. H. T. Bates and T. M. Peters, "Towards improvements in tomography," *New Zealand Journal of Science*, vol. 14, pp. 883, 1971.

[14] R. H. T. Bates, K. Garden, and T. M. Peters, "Overview of computerized tomography with Emphasis on future developments," in *Proceedings of the IEEE*, vol. 71, 1983, pp. 356-372.

[15] L. W. Goldman, "Principles of CT and CT technology," *Journal of Nuclear Medicine Technology*, vol. 35, pp. 115-128, 2007.

[16] J. B. Gui, Z. L. Hu, and Y. Zhou, "Technology evelopment of micro-CT with high spatial resolution," *Computerised Tomography Theory and Applications*, vol. 18, pp. 106-116, 2009.

[17] J. Hsieh, *Computed Tomography: Principles, Design, Artifacts and Recent Advances*, United States of America: SPIE, 2003.

[18] Z. Hu, J. Zou, J. Gui, J. Rong, Y. Li, and H. Zheng, "Geometric calibration method based micro-CT system for small-animal imaging," *IEEE*, vol. 15, pp. 1-4, 2010.

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